General Unification Theorem Proof concerning Functional Dependencies in Relational Databases

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For a problem that was to be solved in my database systems graduate class this last semester, we were asked to prove a theorem by Hugh Darwen (Date, 2003, P. 339) that is known as the General Unification Theorem concerning Functional Dependencies within Relational Databases. Here is the theorem:

If $A \rightarrow B$ and $C \rightarrow D$, then $A \cup (C - B) \rightarrow BD$

This theorem is read as follows: *If A functionally determines B and C functionally determines D, then A union C minus B functionally determines the union of B and D.*

First, I will utilize sets of integers to prove the theorem.
A = {1,2,3,4,5}
B = {3,4,5}
C= {4,5,6,7,8}
D = {5,6,7}

Second, let's determine that each Functional Dependency (FD) is met. Yes, they are met since we see that $A \rightarrow B$ because it is a trivial dependency (the right side is a subset of the left side). We also see that the FD $C \rightarrow D$ is a trivial dependency.

Now we are ready to perform the operations of the theorem on the sets. We first perform the C – B operation as it is a correct "order of operations" procedure. The difference of set C and B here is {6,7,8}.

Next we perform a union of set A with the set {6,7,8} which results in {1,2,3,4,5,6,7,8 }.

Finally, we will see if the FD of the union of A with the difference of C and B functionally determines the union of B and D. BD is equivalent to {3,4,5,6,7}. And yes, we have a proof:

$\{1,2,3,4,5,6,7,8\} \rightarrow \{3,4,5,6,7\}$ as it is a trivial dependency

REFERENCE

Date, C. J. (2003). *An Introduction to Database Systems, Eighth Edition*. Addison Wesley.